MATH 173 **Test 3 (Chapter 4)**

Show all your work in the provided space. No work no credit

1. (8 pts) Use the First Derivative Test to find all relative extrema of the function $f(x) = (x-1)^2(x-3)$

2. (8 pts) The side of a cube is measured to be 3 inches. If the measurement is correct within 0.01, use differentials to estimate the maximum posssible error and relative error in calculating the volume of the cube

Error:

Relative Error:

3. (8 pts.) Find all points of inflection and discuss concavity of the graph of $f(x) = 2x^4 - 8x^3 + 3$ on its domain.

4. (8 pts.) Use the Extreme Value Theorem to find all absolute extrema of $f(x) = x + \cos x$ in the interval $[0, 2\pi]$.

5. (13 pts) Sketch the graph of the function $y = \dfrac{2x+3}{x-4}$.

a) Domain

b) Vertical asymptote

c) Horizontal asymptote

d) Intercepts

e) Relative Extrema

f) Concavity and points of inflection

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6. (7 pts) Show that the function $f(x) = x^3 - x^2 - 2x$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-1, 1]$. Find value of c in $(-1, 1)$ that satisfies $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

7. (6 pts) Find the differential dy of the function $y = x\sqrt{2 + x^2}$.

8. (10 pts) Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3 inches.

9. (14 pts.) Use analytic methods and **First Derivative** Test to find domain, the intervals on which the function $y = x^4 - 32x$ is increasing, decreasing, concave up, concave down. Then find local extreme values and inflection points. Support your answer graphically.

Domain:

Increasing on :

Decreasing on:

Concave up on:

Concave down on:

Local Minimum:

Local Maximum:

Inflection points:

Intercepts:

10. (3 each) Find each limit, if possible.

a) $\lim_{x\to\infty}\left[\ln 2+\ln\left(\frac{x-\sqrt{2}}{x\sqrt{2}}\right)\right]$

b) $\lim_{x\to\infty}\frac{5x^2-2x+3}{4x^2-5}$

c) $\lim_{x\to-\infty}\frac{-3x+1}{\sqrt{x^4+x}}$

d) $\lim_{x\to\infty}\frac{-3(x-2\sin x)}{x}$

e) $\lim_{x\to\infty}\frac{2}{2+e^{-x}}$

f) $\lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)$

11. (5 pts. **Bonus**) Find the point on the graph of $y=\frac{1}{1+x^2}$ where the tangent line has the greatest slope, and the point where the tangent line has the least slope.